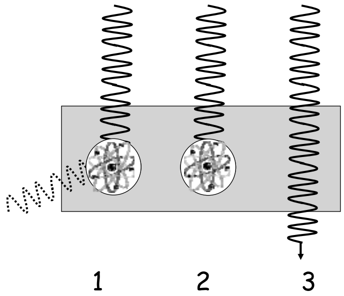
X-ray Interaction (Part I)

Possible Interactions

- Three types of interaction

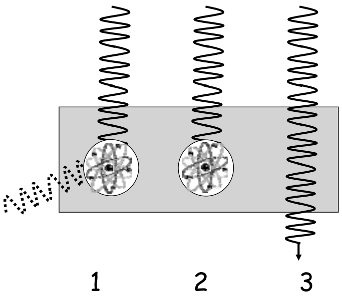
1. Scattering
   - Interaction with an atom
   - Deflected
   - May or may not loss of energy



Possible Interactions

- Three types of interaction

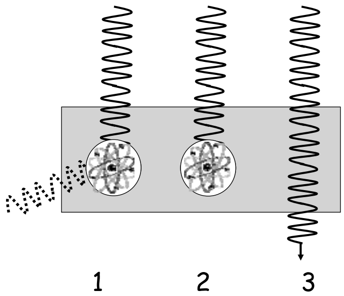
1. Scattering
2. Absorption
   - Interaction with an atom
   - Loses all of it energy to the atom



Possible Interactions

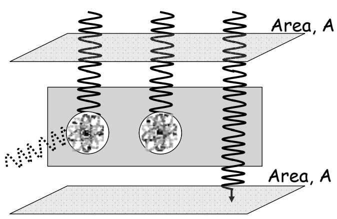
- Three types of interaction

1. Scattering
2. Absorption
3. No Interaction
   - Passes through material without interacting ANY atoms



Possible Interactions

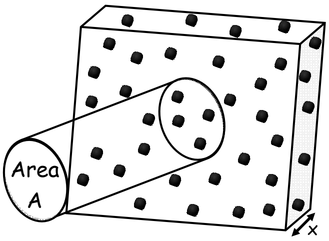
- Compare radiation intensity before and after interaction with medium
- Lower intensity after passing through medium
- Beam has been attenuated by processes of scattering & absorption



Probability & Cross-section

- Probability of X-ray photon interacting with a *particular* atom is low
- However, very large number of atoms in a small volume of solid increase the probability of an interaction

Probability & Cross-section

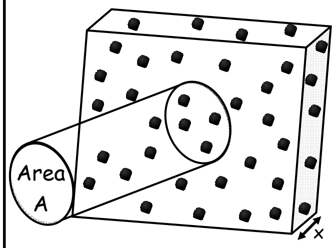


N atoms per unit volume

● Apparent Area, *a*

- X-ray beam of area A incident on medium with atoms of cross-sectional area *a*
- If photon hits atom it is either absorbed or scattered from primary beam
- *Cross-section*
  - Apparent Area, a, NOT the actual area
  - Area likely in interact with X-ray
  - Depends upon Z & photon energy
  - Typically 1.5 X 10^{-28} m^2

Probability & Cross-section



N atoms per unit volume

● Apparent Area, a

Probability of an interaction occurring

- Divide total area of atoms within irradiated area by area of beam

Number of irradiated atoms $= NAx$

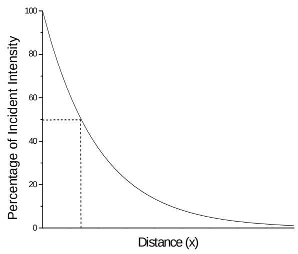
Total area of irradiated atoms $= aNAx$

Probability of intraction $= \frac{aNAx}{A} = aNx$

Total Linear Attenuation Coefficient (μ)

- Parallel beam of monoenergetic photons will undergo exponential attenuation as it passes through a uniform medium

$$I_x = I_0 \exp(-\mu x)$$



- *μ* is the total linear attenuation coefficient
- Definition:
  - *Fraction of X-rays removed from beam per unit thickness of material*
- Increases as probability of interaction (aNx) increases

Total Mass Attenuation Coefficient (μ/ρ)

- Probability of interaction proportional to number of atoms per unit volume, N
- If medium is heated volume will increase and N will decrease
- Example: Double thickness & half density
- μ/ρ will be unchanged

- Definition:
  - *Fraction of X-rays removed from beam per unit thickness of material*

Total Attenuation Coefficient

- Total attenuation coefficient is the sum of the attenuation coefficients due to each attenuation process
  - I.e. total linear attenuation coefficient it the sum of the individual linear attenuation coefficients
  - I.e. total mass attenuation coefficient it the sum of the individual mass attenuation coefficients

Total Attenuation Coefficient

<u>Linear Attenuation Coefficients</u>

- $\mu = \tau + \sigma + \pi$
  - τ linear attenuation coefficient due to photoelectric effect
  - σ linear attenuation coefficient due to Scattering
  - π linear attenuation coefficient due to Pair Production

<u>Mass Attenuation Coefficients</u>

- $\mu/\rho = \tau/\rho + \sigma/\rho + \pi/\rho$

Total Attenuation Coefficient

- Attenuation is the sum of absorption & scattering

$$\mu = \tau + \sigma + \pi$$

Photoelectric	Compton & Rayleigh	Pair Production
$\tau = \tau_a + \tau_s$	$\sigma = \sigma_a + \sigma_s$	$\pi = \pi_a + \pi_s$

Attenuation Processes

Five x-ray attenuation processes:

1. Coherent (or Elastic) Scattering
2. Photoelectric Effect
3. Compton Scattering
4. Pair Production
5. Photodisintegration

Coherent Scatter

- If the energy of a photon is considerably less than binding energies of orbiting electrons of an atom the photon may be deflected from it's path with no loss in energy
- Also called *Classical* or *Rayleigh Scattering*

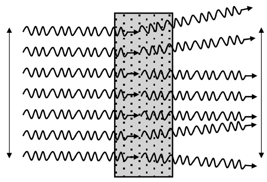
Coherent Scatter

- The photon interacts with an electron, raise it's energy.
- Not sufficient to become excited or ionized
- Returns to original energy level and emits photon with same energy as the incident photon
- Different direction: Therefore scattered

Coherent Scatter

- Predominantly forward scatter
- Elastic scatter can not occur if recoil experienced by atom is sufficient to cause excitation or ionization
- No absorption: No energy has been permanently transferred to material

Coherent Scatter



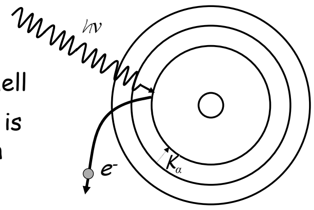
- Low attenuation
- Photons scattered through small angle
- Particularly when E > 100eV & low Z
- Contribution to mass attenuation;

$$\frac{\sigma_{coh}}{\rho} \propto \frac{Z^2}{E}$$

Photoelectric Effect

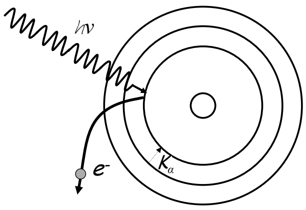
- X-ray photon involved in an inelastic collision with an orbiting electron
- Photon gives up ALL of its energy and therefore disappears (absorbed)
- Electron is ejected from atom
- Absorption can only take place if photon energy is equal to or greater than electron binding energy

Photoelectric Effect



- X-ray photon of energy $h\nu$
- Electron ejected from K-shell
- Some of the photon energy is used in overcoming electron binding energy, B
- Remaining energy is given to electron as kinetic energy
- Electron Kinetic Energy = ($h\nu$ – *B*)

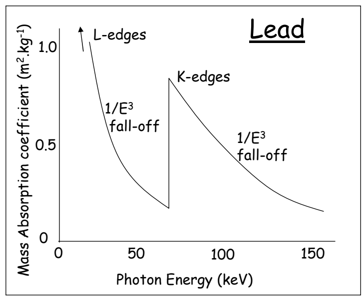
Photoelectric Effect



- Vacancy created in K-shell will be filled by electron from the L-shell
- Quantum 'jumps' producing characteristic radiation
- Energy of characteristic photon is equal to energy difference between shells
- For tissue, energy difference is very small
  - ($1.2 – 1.8 \times 10^{-2}$ eV) => Infrared

Photoelectric Effect

- Probability of photoelectric interaction occurring at a particular shell depends upon binding energy and photon energy
- Zero when
  - $h\nu < B$
- Greatest when
  - $h\nu = B$
- Decreases rapidly with photon energy
  - When $h\nu > B$:



Photoelectric Effect

Photoelectric Effect and Attenuation Coefficient

- Mass attenuation coefficient is related to the atomic number of the absorber (Z) and the photon energy (E)
- Approximated by:

$$\frac{\tau}{\rho} \propto \frac{Z^3}{E^3}$$

- Applies to E up to 200 keV.
- A higher energies E^3 term approximates to E^2 & eventually E

Photoelectric Effect

- Photoelectric Effect causes both *attenuation* and *absorption,* BUT NOT *scattering*
- Individual photons are removed form beam
  - *Attenuation*
- Energy is imparted to the absorbing medium
  - *Absorption*
- Energy absorbed
  - Kinetic energy of ejected photon
  - Energy of recoil of absorbing atom
  - Energy of characteristic radiation

Photoelectric Effect

Contribution of photoelectric effect to the radiographic image

- Need to consider the linear attenuation coefficient:
- For a given photon energy attenuation is proportional to density of tissue and atomic number cubed

$$\tau \propto \frac{\rho Z^3}{E^3}$$

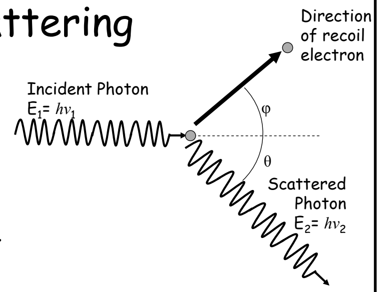
- Bone is approximately twice as dense as soft tissue with an atomic number twice that of tissue
  - Therefore linear attenuation coefficient of bone is approximately 16 times greater than that of soft tissue

Compton Scattering

- If photon energy is much higher than electron binding energy, electron may be considered as a *free electron*
- Interaction between free electron and photon is *Compton Scattering*
- Partial absorption of photon energy

Compton Scattering

- Photon may be scattered in any direction
- Electron can only travel forwards relative to incident photon
- Partial absorption of photon energy



$$\lambda_2 - \lambda_1 = \frac{h(1-\cos\theta)}{mc}$$

Compton Scattering

Compton Scattering and Attenuation Coefficient

- Probability of Compton scattering occurring per unit mass is proportional to the density of electrons and inversely proportional to the photon energy

$$\frac{\sigma}{\rho} \propto \frac{\text{electron density}}{E}$$

Compton Scattering

Electron Density

- Can use Avogadro's number [Cloke Section 5.6] to calculate the number of atoms per mole of an element of mass number (A)

$$\text{atoms per unit mass} = \frac{N_A}{A}$$

- Number of electrons (in a normal atom) is equal to the number of protons (Z)

$$\text{electron density} = N_A \times \frac{Z}{A}$$

Compton Scattering

Electron Density

- Assume that most elements has equal numbers of protons and neutrons
  - Z/A = 0.5
- Hydrogen – 6 X 10^{23} electrons per kg
- Everything else – 2.5-3.5 X 10^{23} electrons per kg

Compton Scattering

Mass Attenuation Coefficient (σ/ρ)

- Inversely proportional to photon energy
- Measure of the total energy removed from the beam
- Sum of scattering & absorption coefficients

$$\sigma = \sigma_a + \sigma_s \quad \text{and} \quad \frac{\sigma}{\rho} = \frac{\sigma_a}{\rho} + \frac{\sigma_s}{\rho}$$

Compton Scattering

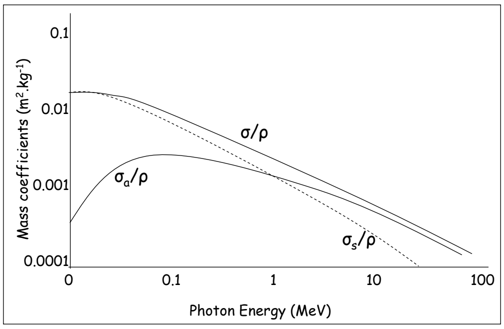
Mass Scattering Coefficient (σ_s/ρ)

- Represents the fraction of total beam energy left to photons

Mass Absorption Coefficient (σ_a/ρ)

- Represents the fraction of the total x-ray beam energy transferred to the medium
- Higher photon energy => higher energy loss
  - σ_a/ρ and σ/ρ are closer together for higher energies

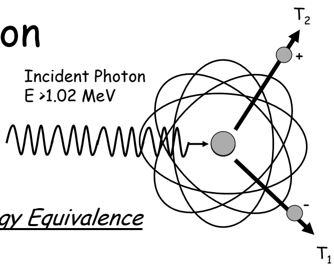
Compton Scattering



Pair Production

- Formation of two charged particles from a single high-energy photon
- Can only occur for photon energies greater than 1.02 Mev
  - (Equivalent to twice the rest mass of an electron)
- Produces electron and positron pair

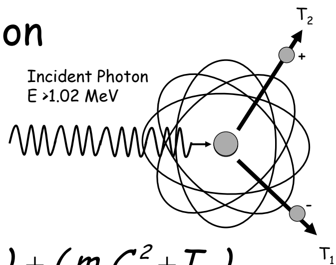
Pair Production



Mass-Energy Equivalence

- [Cloke section 23.4.1]
- Photon interacts with electric field of nucleus
- Photon energy is converted into mass
- Any remaining energy is passed to particles as kinetic energy

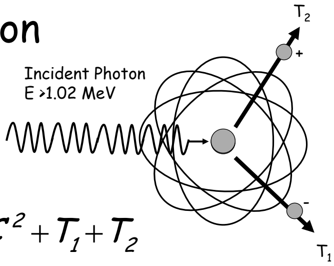
Pair Production



$$E = (m_0 C^2 + T_1) + (m_0 C^2 + T_2)$$

- E is the photon energy
- m_0 is electron (or positron) rest mass
- c is the speed of light
- T_1 & T_2 are the KE of the electron and positron respectively

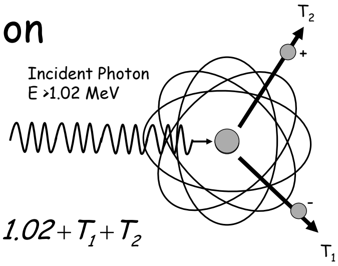
Pair Production



$$E = 2m_0C^2 + T_1 + T_2$$

- E is the photon energy
- m_0 is electron (or positron) rest mass
- c is the speed of light
- T_1 & T_2 are the KE of the electron and positron respectively

Pair Production



$$E(MeV) = 1.02 + T_1 + T_2$$

- E is the photon energy
- m_0 is electron (or positron) rest mass
- c is the speed of light
- T_1 & T_2 are the KE of the electron and positron respectively

Pair Production

Attenuation, Absorption & Scattering

- Attenuation related to both Photon energy & atomic number

$$\frac{\pi}{\rho} = (E - 1.02)Z$$

- KE of electrons & positrons are absorbed by the medium
- Energy absorbed is less than original photon energy
  - (E-1.02) MeV

Pair Production

- Kinetic Energy of electrons & positrons is *absorbed* by the medium
- Energy absorbed is less than original photon energy
  - (E-1.02) MeV
- Electron will eventually loss all it's energy to medium
- Positron will eventually collide with an electron
  - Positron-electron annihilation
  - Producing two photons each with energy 0.51 MeV

Pair Production

- If the two photons of annihilation radiation are absorbed by the medium, then the total energy absorbed is;

$$Energy\ Absorbed = (E - 1.02) + (2 \times 0.51)$$

- I.e. All the original photon energy has been absorbed
- This doesn't always happen!
- In such case absorption coefficient ($\pi_a < \pi$) by fraction (E – 1.02)/E

Pair Production

- If the two particles of annihilation radiation are absorbed by the medium, then the total energy absorbed is; $E_{absorbed} = (E - 1.02) + (2 \times 0.51)$
- I.e. All the original photon energy has been absorbed
- This doesn't always happen
- In such case absorption coefficient ($\pi_a < \pi$) by fraction;

$$\frac{(E - 1.02)}{E} \quad or \quad 1 - \frac{1.02}{E} \quad \Rightarrow \quad \pi_a = \frac{\pi(1 - 1.02)}{E}$$

Pair Production

- As with the previous attenuation processes;

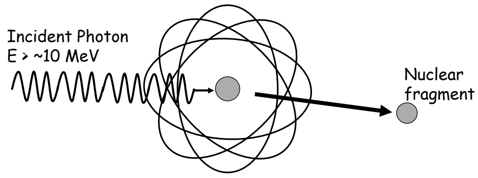
$$\pi = \pi_a + \pi_s \quad \text{and} \quad \frac{\pi}{\rho} = \frac{\pi_a}{\rho} + \frac{\pi_s}{\rho}$$

- π_s is the fraction of energy carried by the two annihilation photons (each of energy 0.51MeV)
- For both diagnostic & therapeutic energies π_s can be ignored

$$\pi = \pi_a \quad \text{and} \quad \frac{\pi}{\rho} = \frac{\pi_a}{\rho}$$

Photodisintegration

- Very-high-energy photons (>10MeV) can escape interaction with electrons and nuclear electric field.



Summary

- Photoelectric effect dominates at low energies (50-500keV)
- Absorption edges are more pronounced for elements with larger Z
- Compton Scattering dominates over a wider range (50keV – 5MeV)
- Compton attenuation is independent of material (with constant density)
- Pair production is only significant for very high energies (>1.02 MeV) and materials with high atomic number (Z)

X-ray Interaction 1 - Problem Sheet

1. When kV is increased, is there an increase or decrease in Compton scattering? Explain your answer
2. A 60 keV x-ray photon ionizes a Barium atom by ejecting an O-shell electron with 12 keV of kinetic energy. What is the energy of the Compton scattered x-ray photon (the binding energy of an O-shell electron is 0.04keV)
3. The energy of the Compton-scattered x-ray is equal to the difference of what two energies?
4. Of the five basic mechanisms of x-ray interactions with matter, which are not important to diagnostic radiography and explain why?
5. A 30 keV x-ray interacts photoelectrically with a K-shell electron of a calcium atom. What is the kinetic energy of the Compton electron? (the k-shell electron binding energy of Calcium is 4 keV)
6. (a) How much less likely will an interaction be for a 50 keV x-ray photon with soft tissue than for a 20 keV photon?
   (b) How much more likely is interaction with iodine than with soft tissue for a 70 keV photon?